

Effective Mathematics Instruction for Learners with Autism Spectrum Disorders

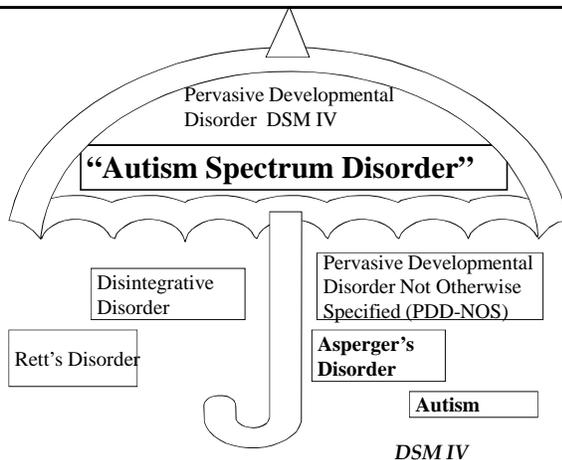
Pennsylvania Training and Technical Assistance Network (PaTTAN)
August 7, 2008
National Autism Conference



Learner Outcomes

Participants will...

- Review seven principles of effective math practice.
- List several learning characteristics of autism and how they may impact performance in math class.
- Review general classroom supports/strategies to assist students in math class.
- Learn specific mathematics strategies for students with Autism Spectrum Disorders.
- Evaluate strategies for mathematics instruction.
- Apply knowledge of effective strategies to evaluate mathematics programs.



**7 Instructional Principles of Effective Practice:
Mathematics Interventions**



**7 Instructional Principles of Effective Practice:
Mathematics Interventions**

1. Instructional Explicitness
2. Instructional Design to Minimize the Learning Challenge

**7 Instructional Principles of Effective Practice:
Mathematics Interventions Cont.**

3. Strong Conceptual Basis
4. Drill and Practice
5. Cumulative Review

**7 Instructional Principles of Effective Practice:
Mathematics Interventions Cont.**

- 6. Motivate to Help Regulate Attention, Behavior, and Hard Work
- 7. On-Going Progress Monitoring

Tips: Teaching Students with Autism Spectrum Disorders

- Provide a predictable environment, try to avoid surprises
- Ensure classroom structure
- Prepare a student for changes in advance
- Teach a strategy for coping for unexpected changes
- Expose a student to a new activity beforehand
- Teach strategies for flexibility of thought
- Minimize transitions
- Allow sensory breaks if needed
- Be aware of what is reinforcing for a student and incorporate those reinforcers into your instructional routine

National Math Panel

- Final Report March 2008
- 22 member panel of experts in the field
- Information related to math instruction for ALL

<http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>

Learning Characteristics of Students with
Autism Spectrum Disorder and Supports
for Math Class



Learning Characteristics:
Language Comprehension in ASD

- Exhibit extremely literal interpretation of language (literal comprehension of directions, idioms, sarcasm, teasing etc.)
 - e.g. "Check your work." may result in student actually drawing a check mark on his paper rather than correcting or reviewing his assignment.
- Difficulty understanding non-verbal communication
- Difficulty in understanding intended messages of others
- Limited understanding of the multiple meanings of words

Learning Characteristics:
Additional Language Challenges

- Initiating and maintaining social interactions
- Perspective taking
 - Difficulty determining the needs, intentions and motives of others
 - Difficulty gauging how to respond to others' needs
 - Difficulty recognizing the expectations of others
 - Difficulty completing obligatory tasks not of their own liking

Deficits in social interaction can negatively impact cooperative learning activities and any unstructured time during the class.

Supports for Math Class

- Be explicit and concrete in your language use
- Teach multiple meanings of math vocabulary
- Check for clarification often
- Carefully consider the impact of social skills deficits when cooperative learning groups are used and plan accordingly

Learning Characteristics: Reading Comprehension in ASD

- Reading comprehension deficits may be associated with:
 - Difficulty understanding abstract language/vocabulary
 - Difficulty in separating essential from non-essential information (e.g. math word problems)
 - Difficulty making inferences

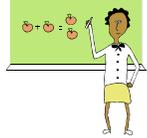
Learning Characteristics: Executive Functioning

- Executive functioning refers to the many skills required to prepare for and execute complex behaviors
 - Planning
 - Organizing
 - Shifting attention
 - Multi-tasking
 - Problem solving
- May manifest as difficulties in representing a problem, making a plan to solve the problem, executing the plan and evaluating the effects of the plan

Supports for Math Class

- Pre-teach vocabulary
- Use of graphic organizers
- Use of rubrics
- Highlight texts or word problems (highlighting tape) to assist student in focusing on most important information

Specific Instructional Strategies Vocabulary Instruction



Explicit Vocabulary Instruction...why?

- Language is the basis of instruction
- We teach through language
- Students build understanding as they process ideas through language
- We diagnose and assess understanding by listening to oral communication and reading math writings
- We don't naturally learn the language of math

Explicit Vocabulary Instruction

Text/Language Issues

“Many texts are written above the grade level for which they are intended.”

Students adopt informal terms as if they are mathematical terms

- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____

Explicit Vocabulary Instruction

Students adopt informal terms as if they are mathematical terms

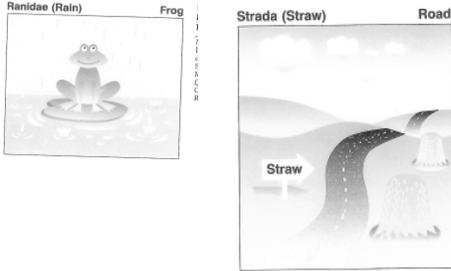
- corner for vertex
- diamond for rhombus
- plus for add
- minus for subtract

Strategies to Teach Vocabulary

Keyword Method

- Used to strengthen the connection between a new word and its associated information
- Three steps to implement:
 - Use similar-sounding keywords
 - Reconstruct abstract information into a symbolic picture
 - Include the definition of the term
 - May incorporate a sentence to tie the keyword to the actual word

Keyword Examples



Keyword Example

- Vocabulary Word: Mode
- Definition: In a set of data, the number that occurs the most
- Keyword: Pie A'la Mode
- Picture: 
- Sentence: Pie A'la Mode is pie with ice cream. It's the best when it has the "most" ice cream.

Some words are shared by mathematics and everyday English, but they have distinct meanings

- | | |
|---------|---------|
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |
| • _____ | • _____ |

Some mathematical terms are found only
in a mathematical context

- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____

Some mathematical terms sound like
everyday English words

- sum/some
- sine/sign
- cosine/cosign
- pi/pie
- dual/duel
- plane/plain
- complement/compliment
- graph/graft
- Plane/plain

Partner Response



Remember!

Visual Supports Should Answer These Questions:

- Where do you want me to go?
- What do you want me to do?
- How will I know when I'm done?
- What do I do then?
- What happens if I do it?
- What happens if I don't do it?
- What happens if I don't know how?

Example

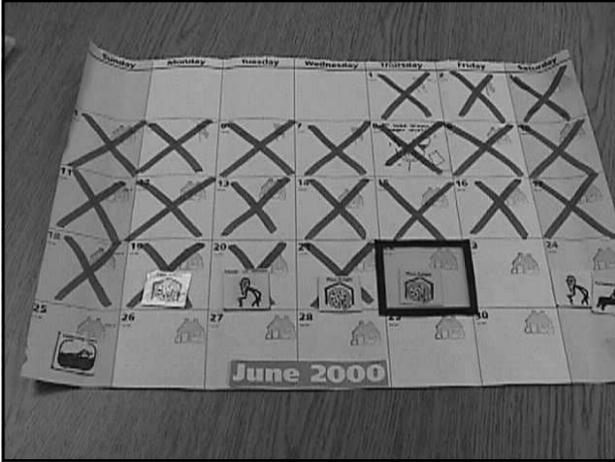
- Sit at desk.
- Take out binder.
- Open to homework section.
- Take out homework assignment and put on my desk.
- Take out my pencil and calculator.
- Look at board and begin to complete warm-up problems.

NAME: _____ ADDRESS: _____

TODAY'S SCHEDULE

DAY: Wednesday DATE: 2/16/00

TIME	#	ACTIVITY	DIRECTIONS FOR ACTIVITY
8:20	X	Personal	
8:34	X	Personal	
8:47	X	Personal Hygiene	
9:17	X	Worktime	
9:25	X	Break	Music
9:30	X	Break	Puzzle
9:39	X	Personal Work	Set up the computer systems
9:48	X	Personal Hygiene	
9:57	X	Break	
10:07	X	Set the Table	Set only one place setting
10:17	X	Break	
10:27	X	Personal Hygiene	
10:37	X	Break	Computer
10:47	X	Wash the Dishes	
10:57	X	Break	Make a base
11:07	X	Personal Hygiene	
11:17	X	Personal	



Sample Visual Supports

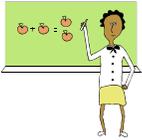
- *Google images* is excellent, but isn't always available through school computers.

Other websites that have excellent, free visual supports:

- www.symbolworld.org has several learning visuals and interactive visual stories.
- www.usevisualstrategies.com printable picture cards are available on this site.
- www.do2learn.com free printable picture cards on a variety of topics, has the capabilities to make visual schedules on this site.
- www.tinsnips.org yes/no boards, response cards, and activity cards all available on this website.



Specific Instructional Strategies
Graphic Organizers



Strategies to Teach Vocabulary

Graphic Organizers

Graphic Organizers

Concept Circles

Graphic Organizers

Frayer Model

Graphic Organizers

List-Group-Label

Graphic Organizers

Verbal/Visual Word Association

Graphic Organizers

Word Sorts

Partner Response



Learning Characteristics of Students with
Autism Spectrum Disorder and Supports
for Math Class



Learning Characteristics :
Writing in ASD

- Fine motor deficits
 - may interfere with efficiently copying problems from board
 - may impact the number of problems that can be completed

Supports for Math Class

- Notetaking
 - Provide student with outline
 - Share peer developed notes
- Structured Study Guides
- Have a student use carbon paper when he/she takes notes
- Have student use Inspiration or other outlining software

Partner Response



Learning Characteristics of Students with Autism Spectrum Disorder and Supports for Math Class



Learning Characteristics : Behavior

- May be extremely “rule bound” and appear rigid and inflexible
- Narrow interests
- Inflexible adherence to routines or rituals
- Repetitive motor movements
- Preoccupation with parts of objects
- Unusual sensory preferences

Supports for Math Class

- Allow for flexibility with sensory issues when using manipulative materials
- Chunk assignments
- Include interests in assignments
- Priming
 - Familiarize student with materials or activity beforehand
 - Introduces predictability and reduces stress
 - Example: Prime with information on index cards
- Modified assignments
 - Allow additional time
 - Highlight texts to maximize reading time
 - Reduce number of items to complete

Partner Response



Empty rectangular box for partner response.

Empty rectangular box for partner response.

Specific Instructional Strategies

CRA

<http://nlvm.usu.edu/en/nav/vlibrary.html>

CRA

Choose the math topic to be taught
Review abstract steps to solve the problem
Adjust steps to eliminate notation or calculation tricks

Match abstract steps with an appropriate concrete
manipulative
Arrange concrete and representation lessons
Teach each type of lesson to mastery
Help students generalize what they learn through word
problems

CRA

- Demonstration and Practice

CRA
Basic Addition

$$\begin{array}{r} 3 + 5 \\ + 3 + 5 \\ + 3 + 5 = +8 \end{array}$$

CRA
Basic Addition

$$\begin{array}{r} + \text{III} + \text{IIII} \\ + \text{IIIIII} \\ \text{So} \\ + \text{III} + \text{IIII} = + \text{IIIIII} \\ + 3 + 5 = + 8 \end{array}$$

CRA
Basic Addition

Problem: $7 + 4$

$$\begin{array}{r} + \text{IIIIII} + \text{IIII} \\ + \text{IIIIIIII} \end{array}$$

CRA
Basic Subtraction

Problem: $9 - 5$

+ $\text{IIIIIIII} - \text{IIII}$

+ $\text{IIIIII} - \text{III}$

+ $\text{IIIIII} - \text{II}$

+ $\text{IIII} - \text{I}$

+ $\text{IIII} - \text{I}$

+ III

CRA: Your Turn

$5 + 2$

$4 + 4$

$4 - 1$

$8 - 6$

CRA: Integers

$- 8 + 3$

- $\text{IIIIIIII} + \text{III}$

- $\text{IIIIII} + \text{II}$

- $\text{IIII} + \text{I}$

- IIII

CRA: Integers

$$\begin{aligned} & - 6 - 4 \\ - & \text{IIIIII} - \text{IIII} \\ - & \text{IIIIIIIIII} \\ - & 6 - 4 = - 10 \end{aligned}$$

CRA: Your Turn

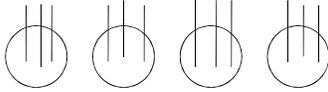
$$\begin{aligned} & 2 - 5 \\ - & 1 - 5 \end{aligned}$$

CRA: Multiplication

- The materials used are both sticks and cups
- **Cups** represent the number of groups
- **Sticks** represent how many are in each group

CRA: Multiplication

4×3
4 groups; 3 in each group



CRA: Your Turn

3×2

1×8

0×12

2×2

CRA: Division

- Materials used are sticks and cups
- Sticks represent the dividend
- Cups represent the divisor

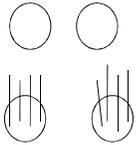
$8 \div 2$
We need 8 sticks and 2 cups

CRA: Division

- Students need to put an equal number of sticks into each cup
- The answer is the number of sticks in ONE cup

$8 \div 2$

IIIIIIII and



CRA: Your Turn

$8 \div 4$

$7 \div 2$

CRA: Simplifying Fractions

The idea is to arrange the sticks into equal groups

$$\frac{3}{9}$$

$$\frac{\text{III}}{\text{IIIIIIII}}$$

CRA: Simplifying Fractions

$$\frac{\text{III}}{\text{III III III}}$$

$$\frac{\text{I}}{\text{III}}$$

CRA: Equivalent Fractions

The idea is the same as multiplying the numerator and denominator

$$\frac{1}{3}$$

CRA: Equivalent Fractions

$$\frac{I}{III}$$

$$\frac{I \quad I}{III \quad III}$$

$$\frac{I \quad I \quad I}{III \quad III \quad III}$$

CRA: Adding Fractions with Unlike Denominators

- Set up the problem with the sticks and slivers of paper as we have been
- The Rule: Multiply (add an additional fraction) for the fraction whose denominator is smaller until you have like denominators.

Example:
 $\frac{1}{2} + \frac{1}{3}$

Unlike Denominator Example

$$\frac{I}{II} + \frac{I}{III}$$

$$\frac{II \quad I}{II \quad II} + \frac{I}{III}$$

$$\frac{I I}{II II} + \frac{I I}{III III}$$

$$\frac{I I I}{II II II} + \frac{I I}{III III}$$

CRA: Simplifying Expressions

- Cups represent the coefficient
- Remember to use the positive and negative signs
- Sticks represent numbers in the expression
- Separate the expression into like parts

Example:
 $-2Y = 3 - Y - 2$

CRA: Simplifying Expressions

$$- \bigcirc \bigcirc Y + III - \bigcirc Y - II$$

CRA: Simplifying Expressions

$$\begin{array}{ccc} & +III & - II \\ - \bigcirc \bigcirc Y & & - \bigcirc Y \end{array}$$

Additional Resources

Strategies and Methods for Instruction



Partner Response







It's QUESTION TIME !!

Resources

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